

# NAG Fortran Library Routine Document

## F01QGF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F01QGF reduces the  $m$  by  $n$  ( $m \leq n$ ) real upper trapezoidal matrix  $A$  to upper triangular form by means of orthogonal transformations.

### 2 Specification

```
SUBROUTINE F01QGF(M, N, A, LDA, ZETA, IFAIL)
INTEGER          M, N, LDA, IFAIL
real           A(LDA,*), ZETA(*)
```

### 3 Description

The  $m$  by  $n$  ( $m \leq n$ ) real upper trapezoidal matrix  $A$  given by

$$A = (U \ X),$$

where  $U$  is an  $m$  by  $m$  upper triangular matrix, is factorized as

$$A = (R \ 0)P^T,$$

where  $P$  is an  $n$  by  $n$  orthogonal matrix and  $R$  is an  $m$  by  $m$  upper triangular matrix.

$P$  is given as a sequence of Householder transformation matrices

$$P = P_m \cdots P_2 P_1,$$

the  $(m - k + 1)$ th transformation matrix,  $P_k$ , being used to introduce zeros into the  $k$ th row of  $A$ .  $P_k$  has the form

$$P_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix},$$

where

$$T_k = I - u_k u_k^T,$$

$$u_k = \begin{pmatrix} \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

$\zeta_k$  is a scalar and  $z_k$  is an  $(n - m)$  element vector.  $\zeta_k$  and  $z_k$  are chosen to annihilate the elements of the  $k$ th row of  $X$ .

The vector  $u_k$  is returned in the  $k$ th element of the array ZETA and in the  $k$ th row of A, such that  $\zeta_k$  is in ZETA( $k$ ) and the elements of  $z_k$  are in A( $k, m + 1$ ), ..., A( $k, n$ ). The elements of  $R$  are returned in the upper triangular part of A.

For further information on this factorization and its use see Section 6.5 of Golub and van Loan (1996).

### 4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

## 5 Parameters

- 1: M – INTEGER *Input*  
*On entry:*  $m$ , the number of rows of  $A$ .  
 When  $M = 0$  then an immediate return is effected.  
*Constraint:*  $M \geq 0$ .
- 2: N – INTEGER *Input*  
*On entry:*  $n$ , the number of columns of  $A$ .  
*Constraint:*  $N \geq M$ .
- 3: A(LDA,\*) – *real* array *Input/Output*  
**Note:** the second dimension of the array  $A$  must be at least  $\max(1, N)$ .  
*On entry:* the leading  $m$  by  $n$  upper trapezoidal part of the array  $A$  must contain the matrix to be factorized.  
*On exit:* the  $m$  by  $m$  upper triangular part of  $A$  will contain the upper triangular matrix  $R$ , and the  $m$  by  $(n - m)$  upper trapezoidal part of  $A$  will contain details of the factorization as described in Section 3.
- 4: LDA – INTEGER *Input*  
*On entry:* the first dimension of the array  $A$  as declared in the (sub)program from which F01QGF is called.  
*Constraint:*  $LDA \geq \max(1, M)$ .
- 5: ZETA(\*) – *real* array *Output*  
**Note:** the dimension of the array ZETA must be at least  $\max(1, M)$ .  
*On exit:*  $ZETA(k)$  contains the scalar  $\zeta_k$  for the  $(m - k + 1)$ th transformation. If  $T_k = I$  then  $ZETA(k) = 0.0$ , otherwise  $ZETA(k)$  contains  $\zeta_k$  as described in Section 3 and  $\zeta_k$  is always in the range  $(1.0, \sqrt{2.0})$ .
- 6: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.  
*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = -1

On entry,  $M < 0$ ,  
 or  $N < M$ ,  
 or  $LDA < M$ .

## 7 Accuracy

The computed factors  $R$  and  $P$  satisfy the relation

$$(R \ 0)P^T = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

$\epsilon$  is the *machine precision* (see X02AJF),  $c$  is a modest function of  $m$  and  $n$  and  $\|\cdot\|$  denotes the spectral (two) norm.

## 8 Further Comments

The approximate number of floating-point operations is given by  $2m^2(n - m)$ .

## 9 Example

To reduce the 3 by 5 matrix

$$A = \begin{pmatrix} 2.4 & 0.8 & -1.4 & 3.0 & -0.8 \\ 0.0 & 1.6 & 0.8 & 0.4 & -0.8 \\ 0.0 & 0.0 & 1.0 & 2.0 & 2.0 \end{pmatrix}$$

to upper triangular form.

### 9.1 Program Text

**Note:** the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      F01QGF Example Program Text
*      Mark 14 Release.  NAG Copyright 1989.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
INTEGER          MMAX, NMAX, LDA
PARAMETER       (MMAX=10,NMAX=20,LDA=MMAX)
*      .. Local Scalars ..
INTEGER          I, IFAIL, J, M, N
*      .. Local Arrays ..
real            A(LDA,NMAX), ZETA(MMAX)
*      .. External Subroutines ..
EXTERNAL        F01QGF
*      .. Executable Statements ..
WRITE (NOUT,*) 'F01QGF Example Program Results'
Skip heading in data file
READ (NIN,*)
READ (NIN,*) M, N
WRITE (NOUT,*)
IF ((M.GT.MMAX) .OR. (N.GT.NMAX)) THEN
    WRITE (NOUT,*) 'M or N is out of range.'
    WRITE (NOUT,99999) 'M = ', M, '    N = ', N
ELSE
    READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
    IFAIL = 0
*
*      Find the RQ factorization of A
CALL F01QGF(M,N,A,LDA,ZETA,IFAIL)
*
    WRITE (NOUT,*) 'RQ factorization of A'
    WRITE (NOUT,*)
    WRITE (NOUT,*) 'Vector ZETA'
    WRITE (NOUT,99998) (ZETA(I),I=1,M)
    WRITE (NOUT,*)
    WRITE (NOUT,*)
```

```

+ 'Matrix A after factorization (R is in left-hand upper triangle)'
  DO 20 I = 1, M
    WRITE (NOUT,99998) (A(I,J),J=1,N)
20  CONTINUE
  END IF
  STOP
*
99999 FORMAT (1X,A,I5,A,I5)
99998 FORMAT (5(1X,F8.4))
  END

```

## 9.2 Program Data

```

F01QGF Example Program Data
  3      5      :Values of M and N
  2.4    0.8   -1.4   3.0   -0.8
  0      1.6    0.8    0.4   -0.8
  0      0      1.0    2.0    2.0   :End of matrix A

```

## 9.3 Program Results

F01QGF Example Program Results

RQ factorization of A

Vector ZETA  
 1.2649 1.3416 1.1547

Matrix A after factorization (R is in left-hand upper triangle)  
 -4.0000 -1.0000 -1.0000 0.6325 -0.0000  
 0.0000 -2.0000 0.0000 0.0000 -0.4472  
 0.0000 0.0000 -3.0000 0.5774 0.5774

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